

Technical Report
853

Optical Anti-aliasing Filters

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13 September 1989

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS



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OPTICAL ANTI-ALIASING FILTERS

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TECHNICAL REPORT 853

13 SEPTEMBER 1989

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ABSTRACT

In an electro-optical imager, a readout device (e.g., CCD or vidicon) samples an image discretely. Often the sampling spatial frequency is much less than the Nyquist rate for the system optics. Scene clutter at higher spatial frequencies than the readout passband is aliased into the band, giving excess clutter in the observed image. A number of designs of pupil plane phase-only spatial filters to reduce this excess clutter, while maintaining the system transmittance, are described. The designs include continuous and binary phase filters in rectangular and polar coordinates. Some of the designs give significant reduction in the out-of-band clutter.

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1. INTRODUCTION

Anti-aliasing filters are used with discrete time sampled electronic systems to prevent signal spectrum foldover (Figure 1). The filters reduce spurious signals due to aliasing and reduce the bandwidth of the noise.

Optical systems with electronic readout typically sample in one dimension (e.g., raster or pushbroom scanning) or two dimensions (CCDs). Figure 2 shows these cases. When the diffraction cutoff spatial frequency of the optics exceeds one-half the sampling spatial frequency of the readout, aliasing occurs. Aliasing introduces false spatial frequencies into the signal (spurious resolution), as shown in Figure 3. If high spatial frequency clutter is present, it can be aliased into the passband of the readout, giving excess clutter in the signal.

The simplest means of preventing aliasing would be to reduce the aperture of the optical system (stop it down) until its cutoff frequency becomes less than half of the readout sampling frequency. Consider a system with a CCD readout with a sampling increment of $25\text{ }\mu\text{m}$, for use with visible ($0.55\text{-}\mu\text{m}$ wavelength) light. To prevent aliasing completely, the system f/number would have to be greater than 90. A more practical compromise would be $f/45$, giving some aliasing but increased signal per pixel. Depending on the number of pixels in the readout (which may be limited by data handling rates or detector technology), the need for low light level performance, and the desired angular field coverage of the sensor, a high f/number may be unacceptable.

This study is to examine the possibility of making phase-only modifications to an existing optical system that would give an anti-aliasing effect without reducing the throughput of the optics. The goal is to achieve the highest possible incoherent modulation transfer function (MTF) in the passband of the readout, with the MTF being as close to zero as possible everywhere outside the passband. Recall that the MTF is the modulus of the optical transfer function (OTF), which is equal to the scaled complex autocorrelation of the pupil function of the optics. Because of the autocorrelation, a phase-only mask can have the desired properties.

Because the phase (in waves or radians) introduced by such a mask is inversely proportional to wavelength, the spectral band of operation of the mask is important. The masks to be studied below are intended for operation over the bands $500 \pm 15\text{ nm}$ (Band 1) and $600 \pm 5\text{ nm}$ (Band 2).

Several different types of phase-only masks have been examined by numerical calculation. Some of these masks will be made and tested in the laboratory. No single design has been identified as ideal for all applications, so the virtues and drawbacks of each type will be discussed.

2. COMMON CONSIDERATIONS

2.1 SAMPLING WITH DETECTORS OF FINITE SIZE

In a real optical system, the detectors in the readout have finite area; they are not point samplers. Assume all the detectors are identical, and that each detector responds as a function of position; call this $r(x,y)$. Assume also that the detectors are uniformly spaced on a rectangular grid of spacings Δx and Δy . Then, the sampling process can be represented mathematically as follows: take the cross correlation of the detector spatial response function with the image, and sample that at $(\Delta x, \Delta y)$. In the Fourier transform plane, the correlation becomes a product with the flipped transform of the response function; call this $R(-\xi, -\eta)$. If the response function is a square of size Δx , and $\Delta y = \Delta x$ (100% fill factor), then a one-dimensional representation of the spectrum of the sampling process is as shown in Figure 4. The finite detector size effects some prefiltering that reduces the aliasing.

2.2 CLUTTER REDUCTION FIGURE OF MERIT

The clutter in the scene will be assumed to have a uniform power spectral density (PSD) in order to give white spatial noise. Total noise power is then proportional to the integral over all spatial frequencies of the *square* of the product of the MTF of the optics and the two-dimensional sinc function from the finite detector size. (The square is taken to obtain the PSD for the image; this is the correct PSD definition for an incoherent imaging system.) The figure of merit is defined as the quotient of the integral of the PSD of a diffraction limited optical system (including the finite detector size) *outside the desired passband of the system* and the integral of the out-of-band PSD of the system plus the anti-aliasing filter. For two-dimensional symmetrical sampling, the desired passband of the system is a circle in frequency space, centered at the origin, and of diameter $1/\Delta x$ in both axes (cutoff at radius $1/2\Delta x$). Figure 5 represents this pictorially. Note that if the anti-aliasing filter is perfect (its PSD is zero outside the passband), the figure of merit goes to infinity. The figure of merit is always greater than or equal to 1.

Note that this figure of merit does *not* include clutter in the passband of the system readout. The figure of merit also does not include any consideration of the reduction in the PSD of the *signal*, so it is easy to define systems with little aliased clutter that give poor imagery. Other criteria (e.g., having the MTF be approximately equal to that of a stopped-down system) must be used to retain contrast. The figure of merit is only a measure of the effectiveness of the system in rejecting out-of-band clutter. A figure of merit of 1 means that the system has no better rejection than a diffraction limited system of the same aperture; a figure of merit of infinity means that no better rejection is possible.

3. FILTER DESIGNS

3.1 GOLAY CODES FOR ONE-DIMENSIONAL FILTERING

If the image readout in the optical system samples in one dimension, as with a raster or pushbroom scan system, a one-dimensional anti-aliasing filter may be all that is needed. If electronic time-domain sampling is to be done in the scan direction, electronic filtering can be used to prevent aliasing in that dimension. (For a pushbroom scan system, this may require one filter per scan line.) One-dimensional filtering can be performed with a binary phase mask.

Golay codes¹ are binary codes. A complementary pair of Golay codes of length N have the property that the sum of the autocorrelations of the two codes is equal to $2N$ times the autocorrelation of a single "chip" (Figure 6), where a chip is the smallest element of the code. There are Golay code pairs for any N that is a power of 2.

Figure 7 shows one scheme for using one-dimensional Golay codes for filtering. The optics' aperture of diameter D is clipped to a square of side S ($S \leq D/\sqrt{2}$). The square is then divided into sections of width $S/4$. Sections 1 and 3 are made to be binary phase masks that are Golay complements, with the correct half-wave depth for Band 1. Sections 2 and 4 are made Golay complements of the correct depth for Band 2. (Spectral filters prevent crosstalk between the pairs of sections.) Figure 8 shows a surface plot of the OTF of a mask with Golay codes of length 4. Note that the scale of the autocorrelation is such that the cutoff frequency for the square aperture of side S would be S/lf , where l is the wavelength, and f is the focal length of the optical system. Note also that a mask of this form gives a band of spatial frequencies where the MTF is zero beyond the passband.

The disadvantage of this technique is that the aperture area is cut to $1/\pi$ (≈ 0.318) that of the full circular aperture per wave band. (The throughput remains much greater than for a stopped-down aperture, particularly if the readout passband is much less than the cutoff spatial frequency for the aperture.) The method is also intended for one-dimensional use, although the secondary maxima in the MTF are much less than the central peak.

A two-dimensional clutter figure of merit is defined above. The similar one-dimensional quantity (where the integrals are taken only along the spatial frequency axis corresponding to the direction along the code) for the Golay code of length 8 is infinity at the design wavelength. The reference MTF was for a diffraction limited aperture width of S .

The Golay masks may also be extended to two dimensions. Figure 9 shows a two-dimensional mask generated by taking two, length 16, one-dimensional codes, and writing them in sequence along rows into a 2 by 8 array. (In general, if a pair of codes of length $N = 2^{m+n}$ are written into a pair of arrays each 2^m by 2^n , then the sum of the autocorrelations of the resulting arrays has the Golay property in two dimensions.) Shown in Figure 10 is a surface plot of the autocorrelation of the mask in Figure 9.

¹ M. J. E. Golay, IRE Trans. Inform. Theory, IT-7, April 1961, pp. 82-87.

3.2 SQUARE BIPHASE MASKS FOR TWO-DIMENSIONAL FILTERING

Figure 11 shows a square biphas mask that is designed to be a two-dimensional anti-aliasing filter. The mask was created by using a global search technique to find a pattern with minimal secondary correlations close to the central peak. (This method would be impractical for masks much larger than 4 by 4.) Figure 12 is a surface plot of the OTF produced for this mask. The largest absolute value of a secondary correlation is 3/16 (the peak is 1), and it occurs at high spatial frequencies.

3.3 ROTATIONALLY SYMMETRIC CONTINUOUS PHASE MASKS

A rotationally symmetric phase function may be placed on an optical surface by conventional figuring techniques or with a diffractive optic. The rotationally symmetric phase function here was taken as a radial polynomial including orders 2, 4, 6, 8, and 10.

The coefficients of the radial polynomial were determined by an optimization procedure. The goal of the optimization was to give a best match (in the rms error sense) to the MTF of a reduced aperture. (An alternate view of this method is generating a phase-only hologram in the system pupil that approximately gives the same point spread function as a smaller circular aperture.) The figure of merit defined above was not used in the optimization because it would tend to give a low MTF within the passband.

Figure 13 shows the monochromatic MTF for a continuous phase function that is intended to give a passband that cuts off at one-tenth the diffraction limited cutoff of the full aperture. It has a figure of merit of 423.8. The figure of merit for a reduced aperture would be infinite. Figure 14 gives the phase function itself, and Figure 15 shows the residual phase after the best-fit focus error of 2.181 waves is removed. (Note that this is *not* the same as setting the second-order term in the polynomial to zero; the radial polynomial terms are not orthogonal.)

The dominant term in the radial phase function is the focus error. In practice, only the fourth- and higher-order terms in the polynomial would be needed, with the quadratic term supplied by defocusing the system. Unfortunately, this implies that the MTF would then be sensitive to focus errors and system aberrations. In a real system, this may be a serious limitation.

3.4 ROTATIONALLY SYMMETRIC BINARY PHASE MASKS

For the continuous radial phase function above, the *sign* of the phase error is unimportant; changing the sign does not change the resulting MTF. A biphas (binary phase) mask is a diffractive optical element that has phase values of zero or π only (alternately, it has amplitude values of +1 or -1).

If the mask is generated from a continuous phase function by taking the cosine of the phase and setting all nonnegative values to +1, and all negative values to -1, then the mask has the following properties:

- (a) All of its even diffractive orders (including the zero order) are absent;
- (b) The phase for each diffractive order is equal to the product of the order number and the original generating phase function;

- (c) As a consequence of (b), adding a constant phase to the generating phase function changes the MTF of the system, because this leads to a constant phase in each diffracted order that is equal to the product of the added phase and the order number.

The idea behind using a biphasic mask is to produce a similar effect as using a continuous phase mask, but because both positive and negative orders are present, the sensitivity to focus shifts may be less (i.e., putting the system in focus for the +1 order would cause it to be out of focus for the -1 order). It was hoped that sensitivity to aberrations other than focus would also be lower than with the continuous phase function.

The generating phase function for the biphasic mask was computed by the same sort of optimization as used above for the continuous phase mask, but the radial polynomial included orders 0, 2, 4, 6, and 8 only.

Figure 16 shows the MTF for a system designed for a passband of width one-tenth of the diffraction limited system cutoff frequency. It has a figure of merit of 587.9. Figure 17 is a two-dimensional representation of the mask: white is phase 0, black is phase π , and gray is zero transmittance. Figure 18 gives the phase generating function. Because the dominant term is of the second order, the mask is nearly the same as a biphasic Fresnel zone plate.

3.4.1 Sensitivity of Biphasic Mask to Wavelength

A rotationally symmetric biphasic mask was computed for a wavelength of 542 nm. It gave a figure of merit (as defined above) of 8.56. At the upper extreme of Band 1 (615 nm) the optical path step of the mask was in error (with respect to 0.5 waves) by approximately -0.06 waves, giving a merit function of 319.6. At 485 nm (the short wavelength extreme of Band 1), the error was +0.06 waves, and the figure of merit was 324.8. The MTFs at the different wavelengths were nearly identical to the MTF at the design wavelength. The performance of the biphasic mask appears to be adequate over the two wavelength bands if the design wavelength of the mask is chosen correctly.

3.4.2 Sensitivity of Biphasic Mask to Aberrations

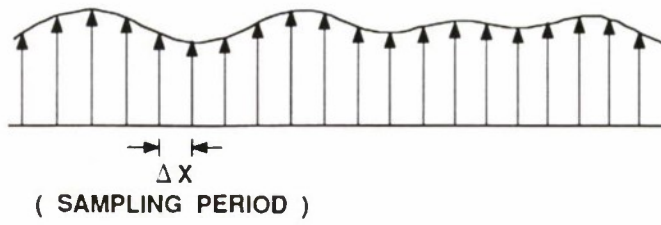
Figure 19 gives the MTF of the above rotationally symmetric biphasic mask with one wave of focus error added. Its corresponding figure of merit was 18.0. This shows that the biphasic mask is relatively insensitive to aberrations in the optical system whose MTF is modified by the mask.

4. SUMMARY

Several different approaches were examined to provide an optical anti-aliasing function in a sensor system that minimizes the amount of light that must be discarded to reduce the clutter signal. The optimum approach has not been identified, but binary phase masks appear to provide a practical approach to the problem.

Further analysis needs to be performed for a real sensor, with known optical and readout characteristics. The addition of a single binary optical element (or binary surface on an existing element) can significantly improve the clutter rejection of a sensor.

SAMPLED
FUNCTION



REPLICATED
SPECTRA

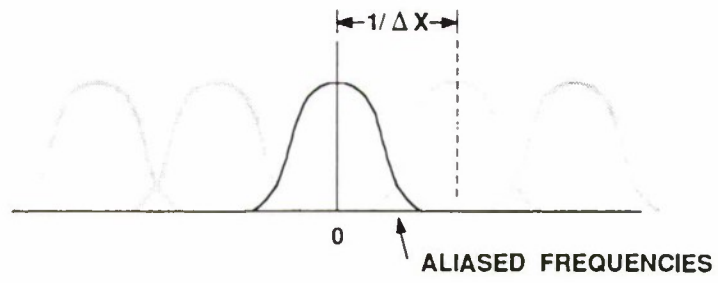


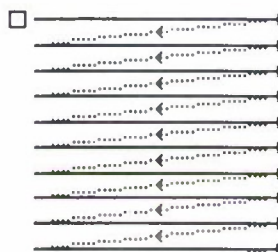
Figure 1. Aliasing with a one-dimensional signal.

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PUSHBROOM
(1-D Array,
1-D Sampling)



RASTER
(Single Detector,
1-D Sampling)



2 - D ARRAY
2 - D SAMPLING

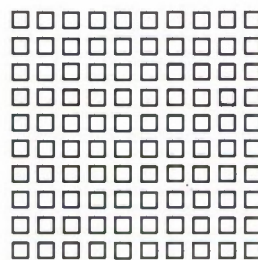
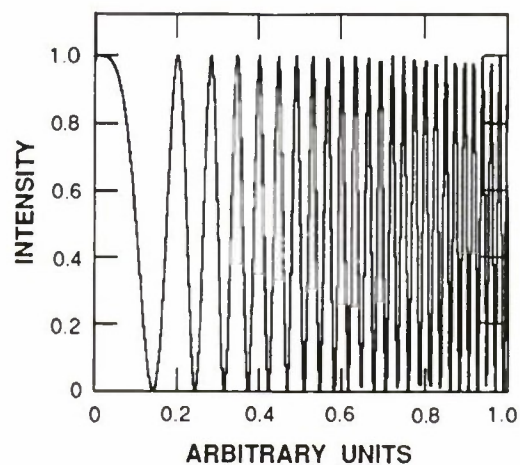
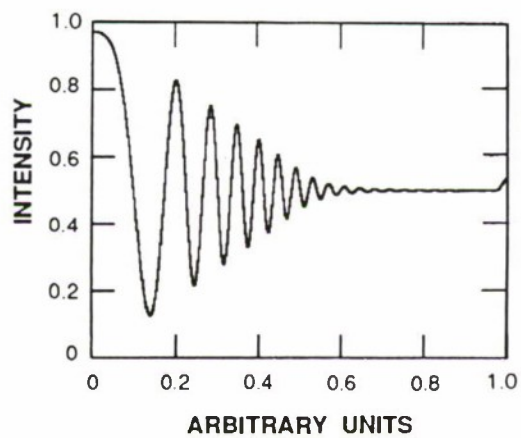


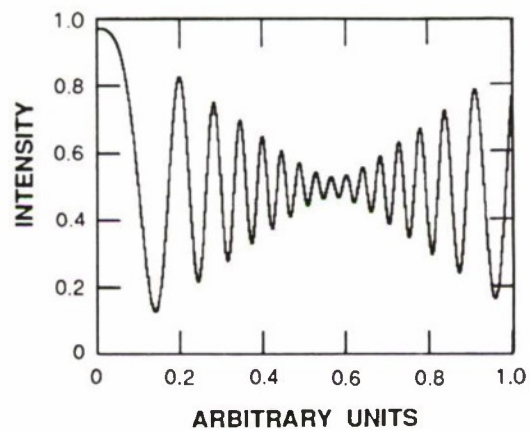
Figure 2. Typical optical readout techniques.



(a)



(b)



(c)

Figure 3. (a) Chirp grating, (b) filtered chirp grating, and (c) sampled then filtered chirp grating, showing aliasing.

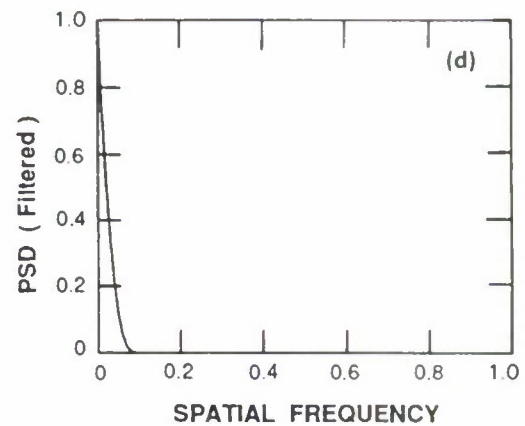
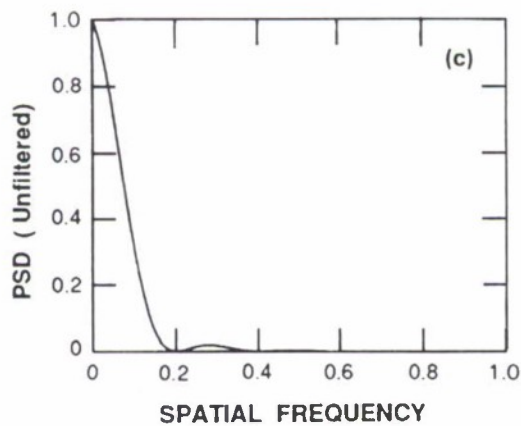
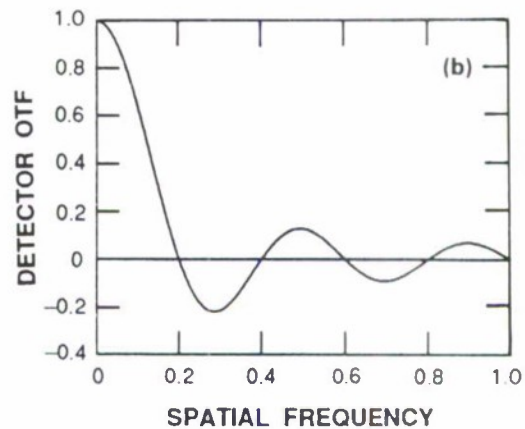
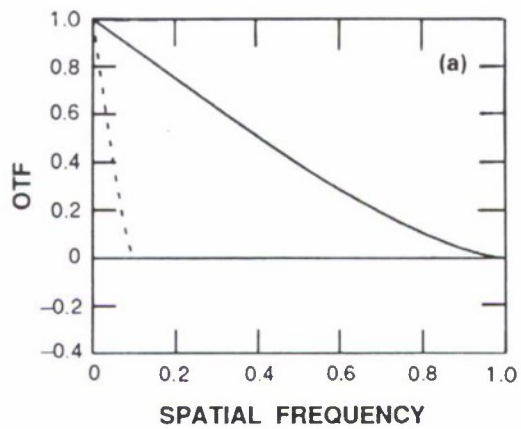


Figure 4. (a) MTF of large aperture (solid) and small aperture(dashes), (b) OTF of finite detector, (c) MTF of large aperture plus finite detector, and (d) MTF of small aperture plus finite detector.

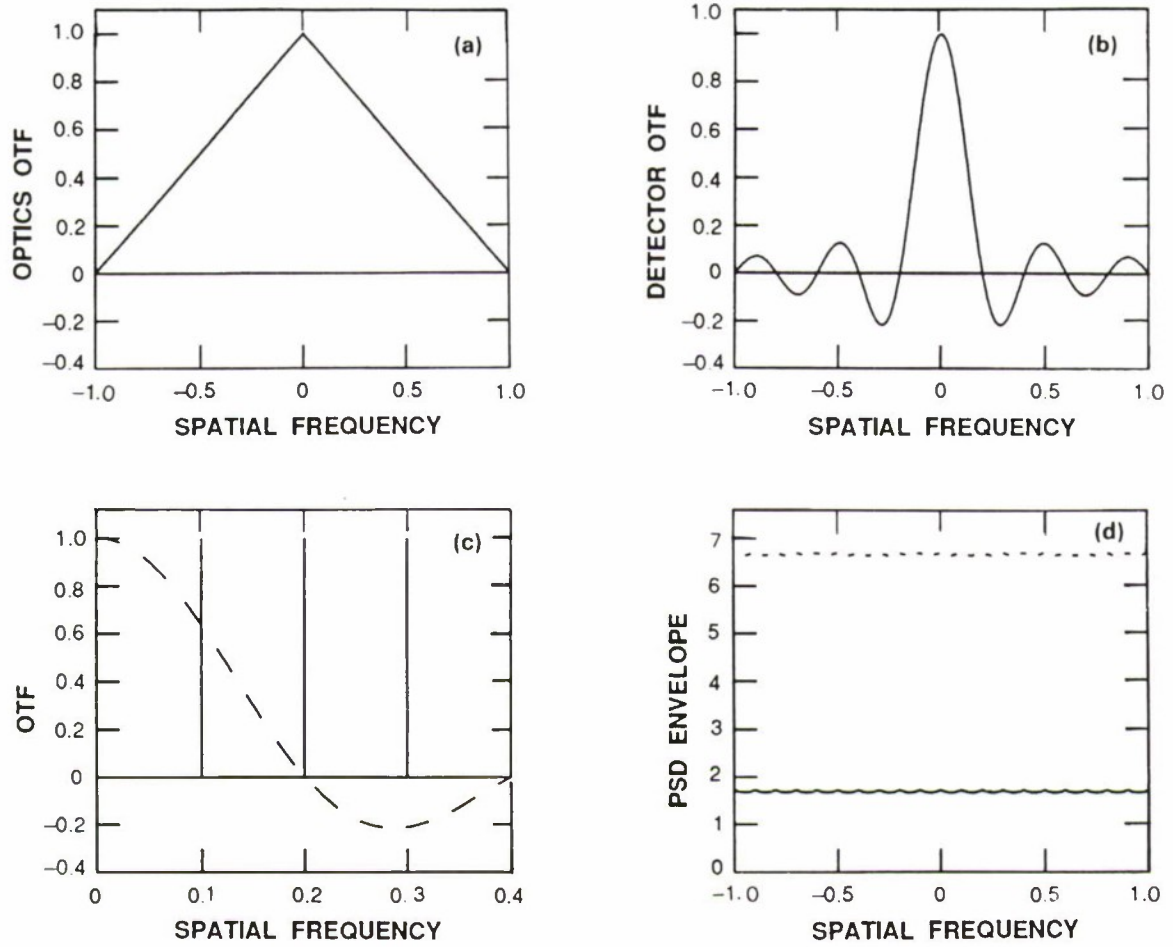


Figure 5. (a) MTF of one-dimensional optics, (b) OTF of finite detector, (c) sampling due to detector spacing (solid), and detector OTF (long dashes), and (d) PSD envelope with sampling: diffraction limited (dashes), diffraction limited plus finite detector size (solid).

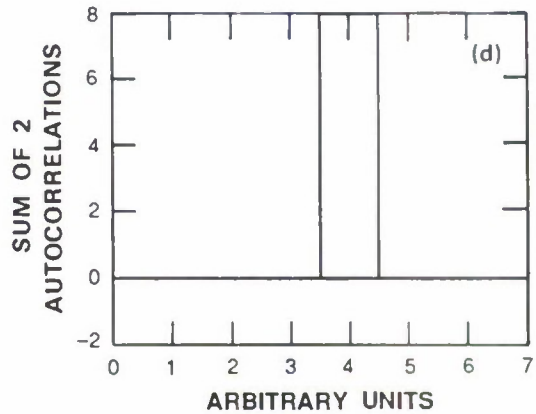
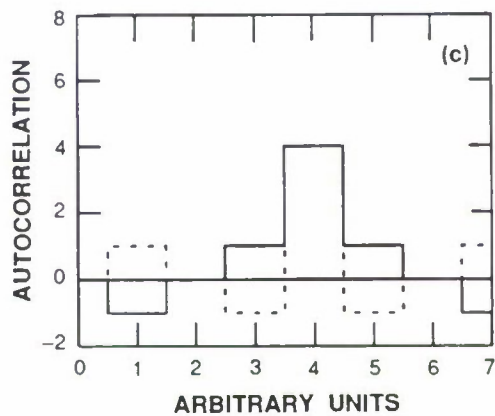
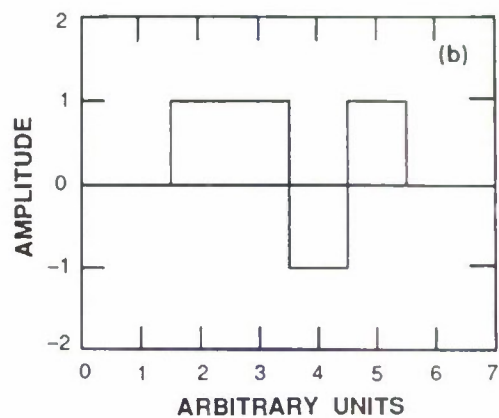
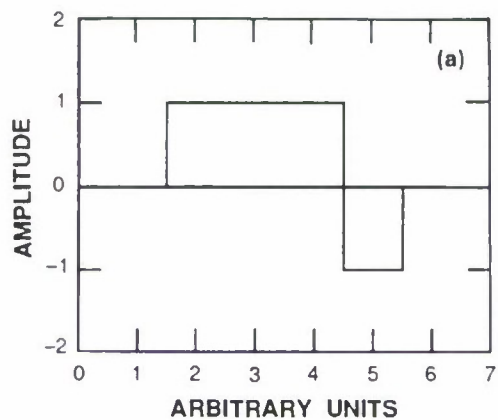
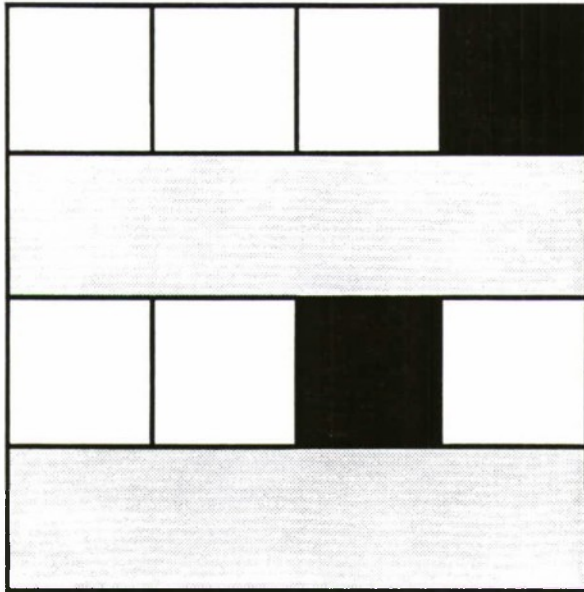
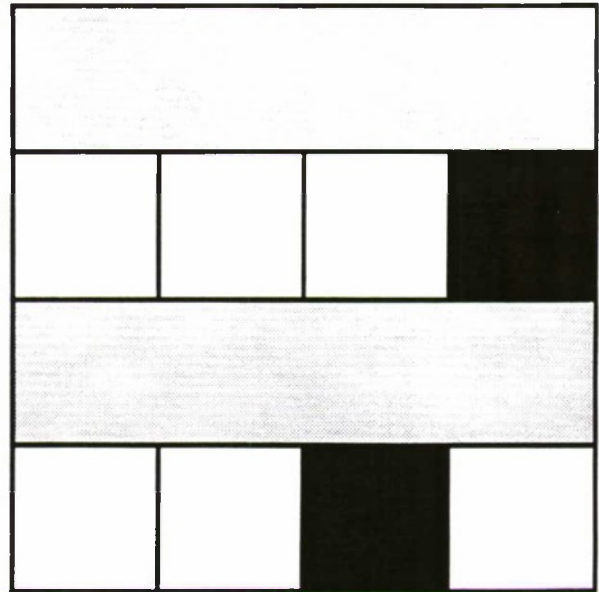


Figure 6. (a) Golay code of length 4, (b) complementary Golay code, (c) autocorrelation of Golay code (solid) and complement (dashes), and (d) sum of autocorrelations.



BAND 1



BAND 2

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Figure 7. Golay masks for one-dimensional filtering.

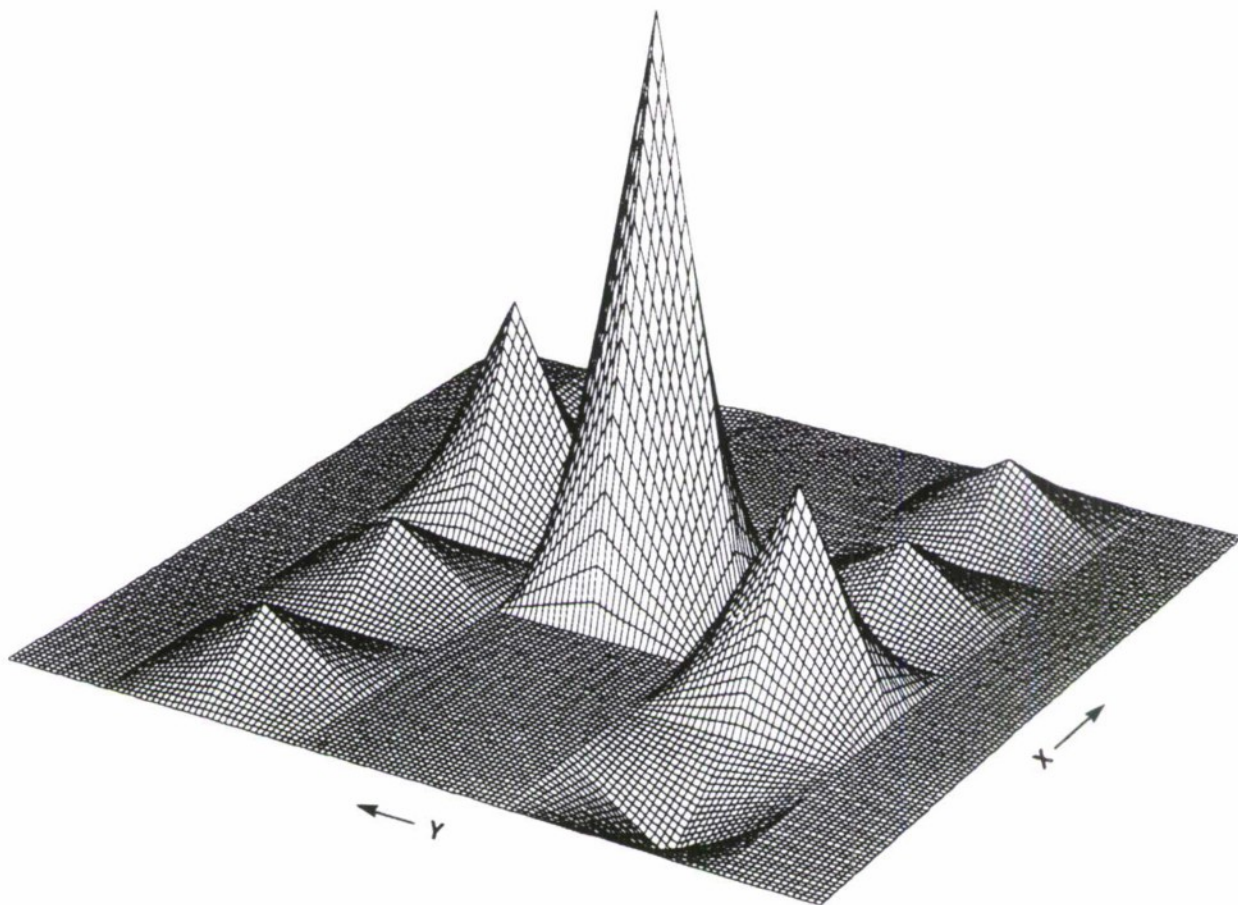
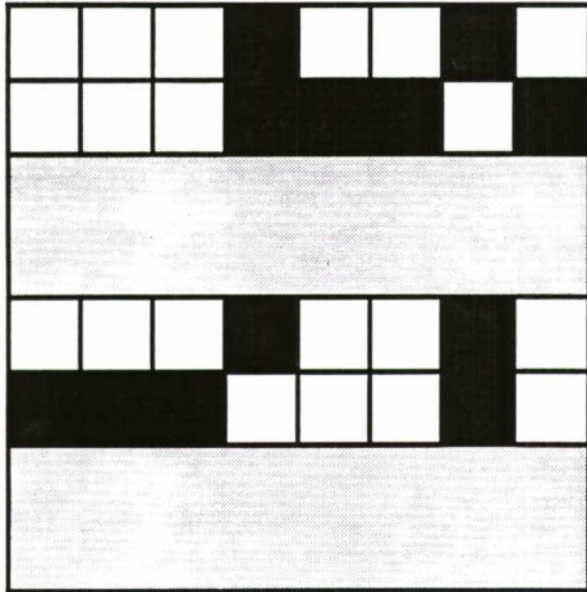
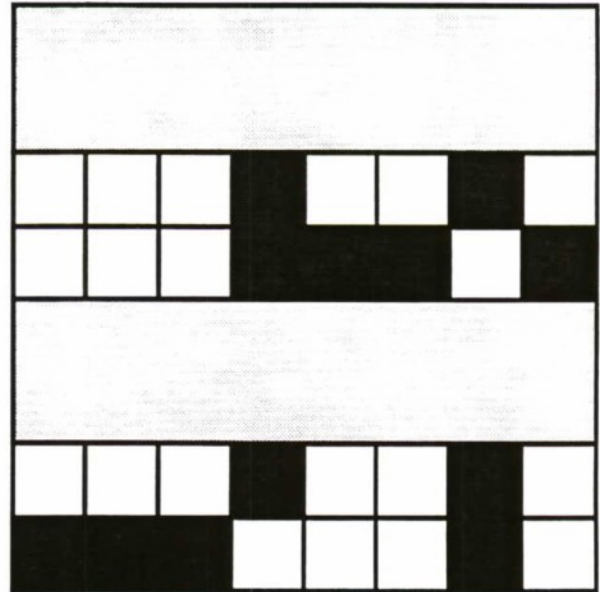


Figure 8. Surface plot of OTF of Golay mask.



BAND 1



BAND 2

125928-9

Figure 9. Two-dimensional Golay masks.

125928-10

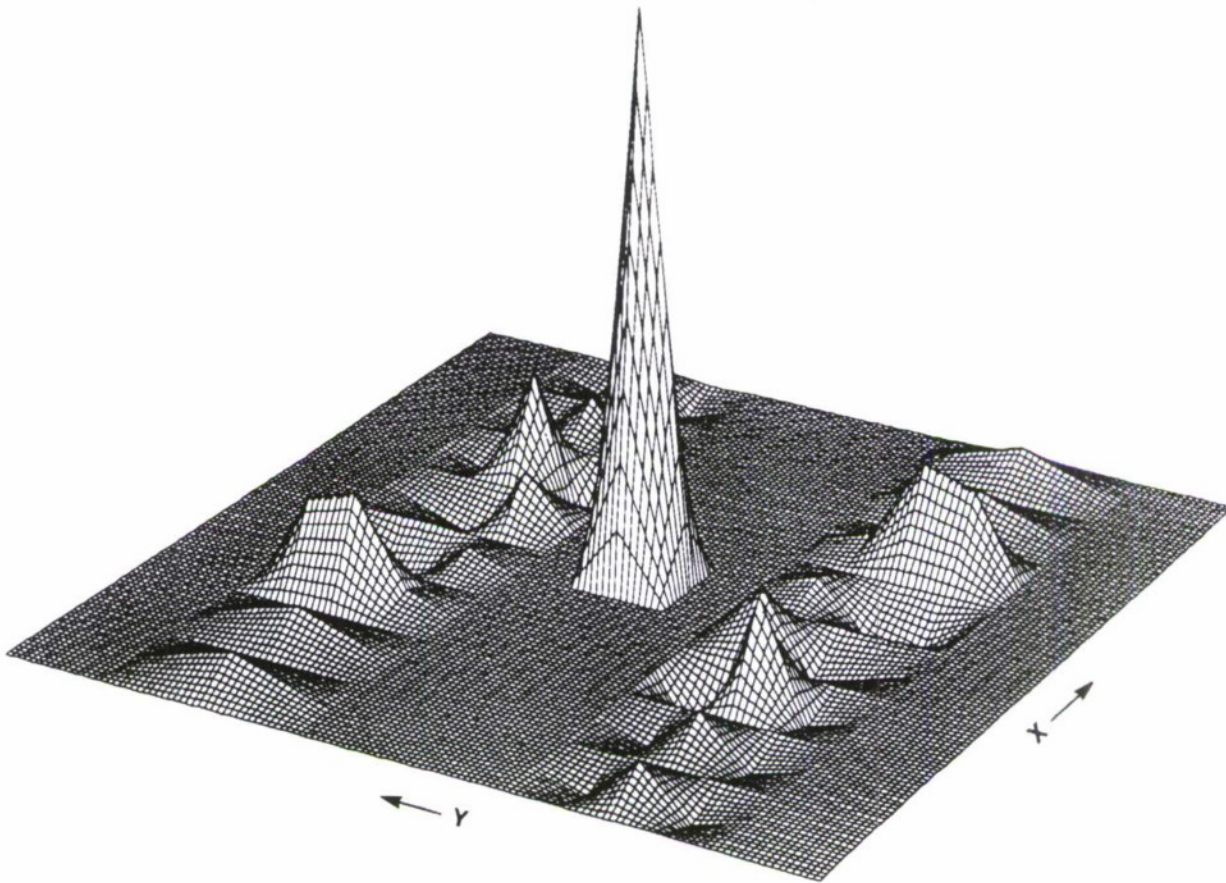


Figure 10. Surface plot of OTF of Golay mask shown in Figure 9.

125928-11

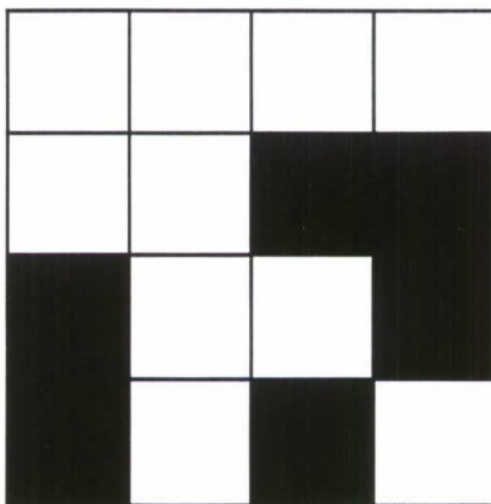


Figure 11. Square binary mask for two-dimensional filtering.

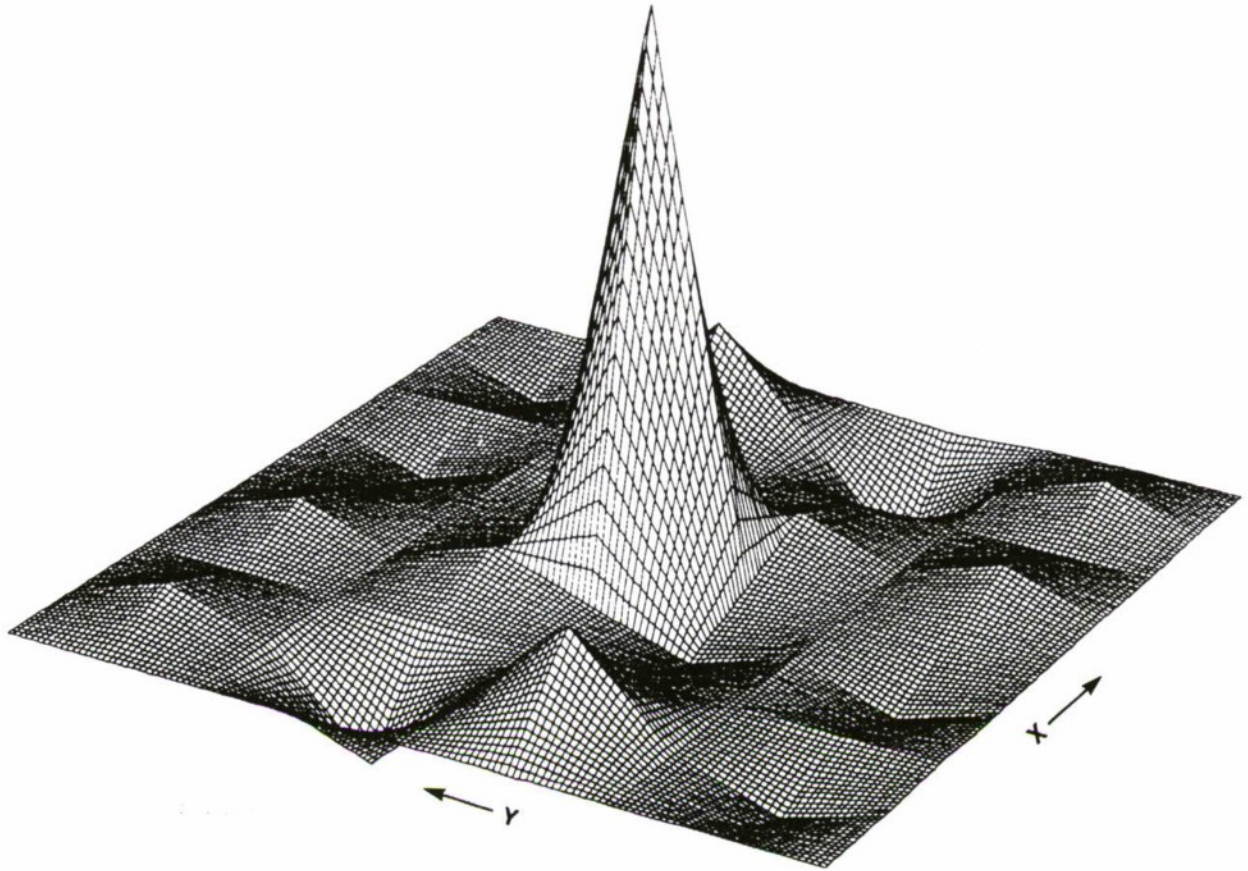


Figure 12. Surface plot of OTF of mask shown in Figure 11.

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125928-13

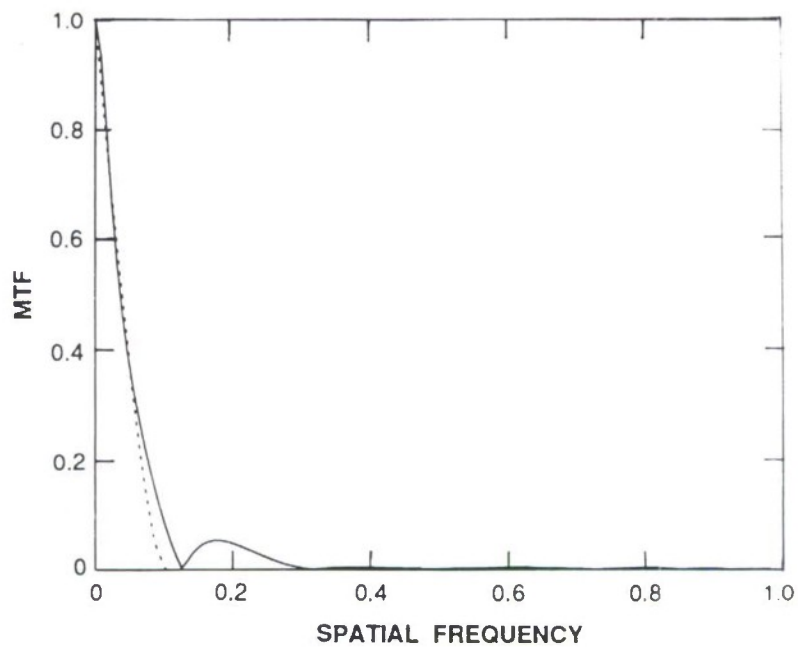


Figure 13. MTF of continuous phase mask (solid) and small aperture (dashes).

125928-14

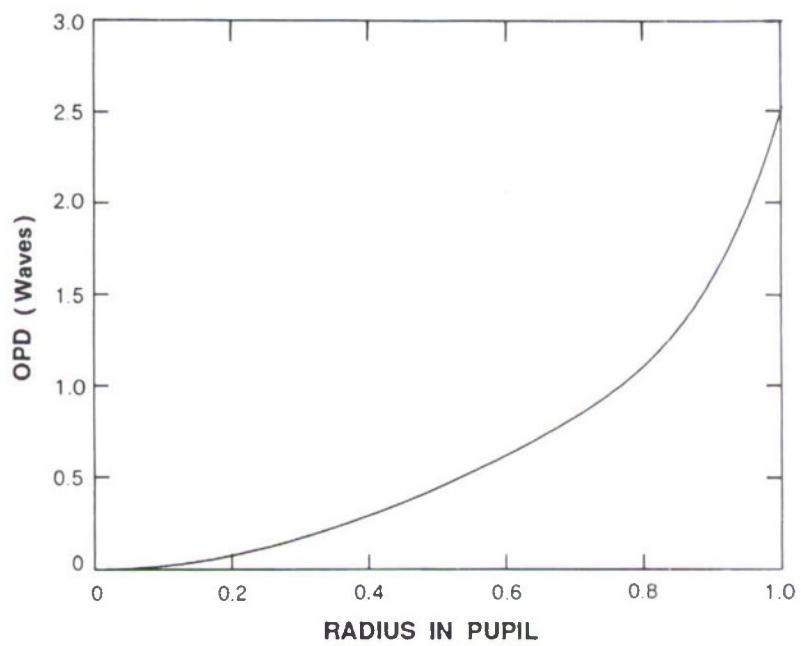


Figure 14. Radial continuous phase function.

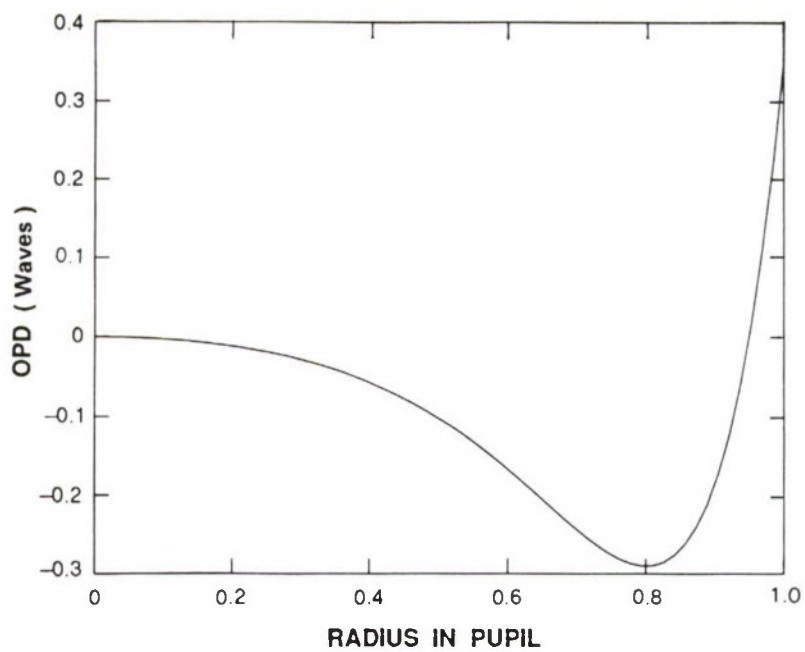


Figure 15. Radial phase function after removal of best-fit rms focus.

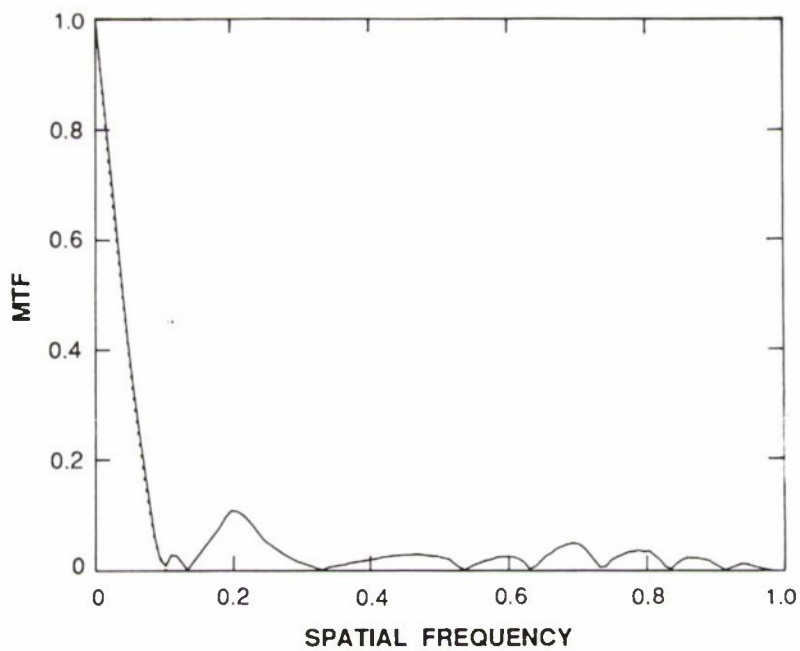


Figure 16. MTF for a rotationally symmetric biphasic mask (solid) and small aperture (dashes).

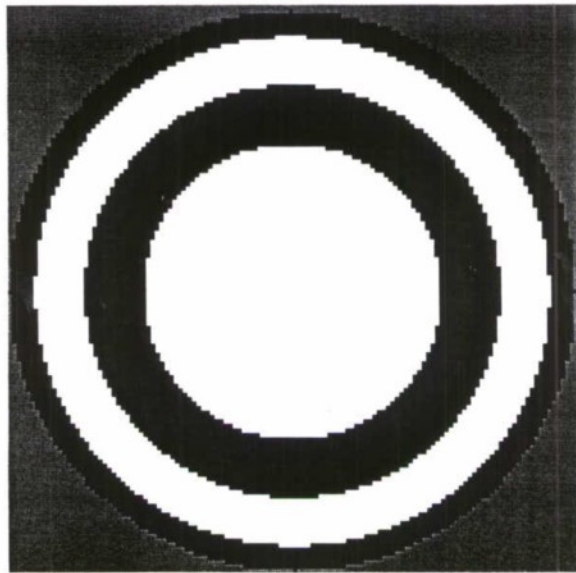


Figure 17. Representation of a biphase mask: 0 phase (white), π phase (black), and 0 amplitude (gray).

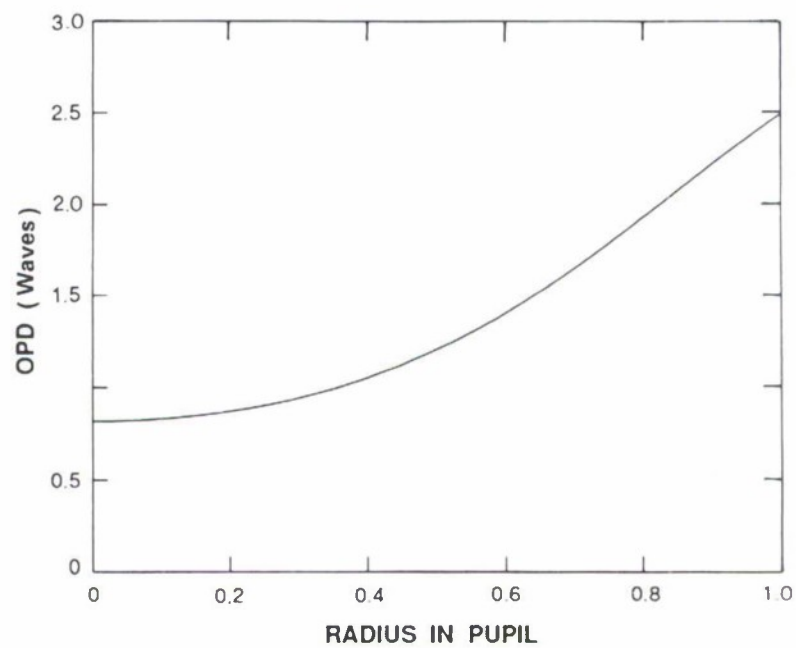


Figure 18. Generating function for biphase mask.

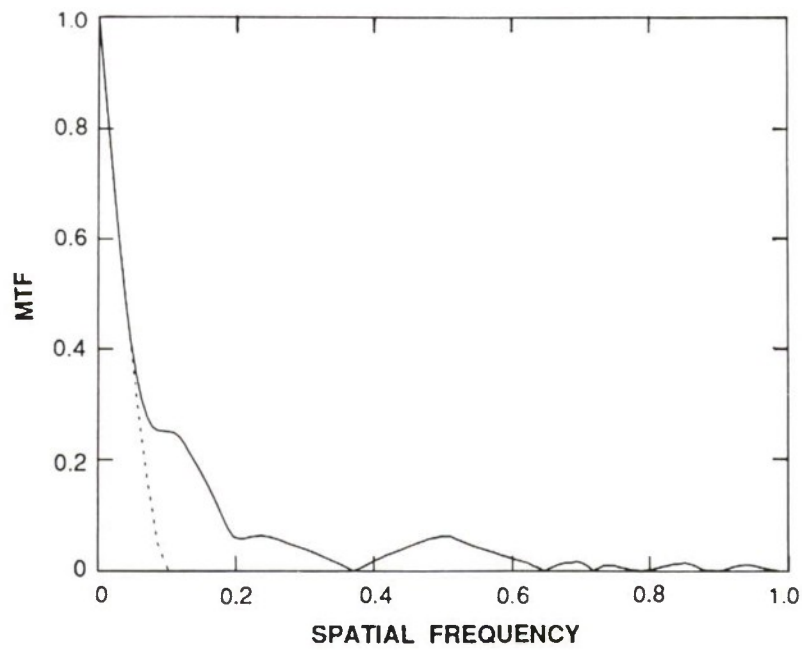


Figure 19. MTF of biphase mask plus one wave of focus error.

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<p>In an electro-optical imager, a readout device (e.g., CCD or vidicon) samples an image discretely. Often the sampling spatial frequency is much less than the Nyquist rate for the system optics. Scene clutter at higher spatial frequencies than the readout passband is aliased into the band, giving excess clutter in the observed image. A number of designs of pupil plane phase-only spatial filters to reduce this excess clutter, while maintaining the system transmittance, are described. The designs include continuous and binary phase filters in rectangular and polar coordinates. Some of the designs give significant reduction in the out-of-band clutter.</p>						
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